Examiners' Report:
Final Honour School of Mathematics Part A
Michaelmas Term 2021

January 6, 2022

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2021	2020	2019	2018	2017	2021	2020	2019	2018	2017
70–100	53	43	57	57	57	37.32	32.58	35.19	35.62	36.77
60–69	57	65	71	69	62	40.14	49.24	43.83	43.12	40
50–59	29	21	27	22	31	20.42	15.91	16.67	13.75	20
40–49	2	3	5	9	4	1.41	2.27	3.09	5.62	2.58
30–39	0	0	1	3	1	0	0	0.62	1.88	0.65
0–29	1	0	1	0	0	0.7	0	0.62	0	0
Total	142	132	162	160	155	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.
- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.
- **Numbers taking each paper.**
  All 142 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	142	30.78	9.73	67.58	14.8
A1	142	37.63	6.61	66.09	10.14
A2	142	60.88	17.08	66.51	10.69
A3	84	34.13	10.53	66.14	14.4
A4	118	29.47	9.24	66.54	11.79
A5	83	35.61	9.7	66.6	13.28
A6	72	35.11	7.05	66.06	11.77
A7	52	27.02	8.9	65.25	11.4
A8	127	25.5	8.1	66.3	10.81
A9	85	30.32	9	66.34	11.5
A10	36	40.14	6.61	66.25	13.54
A11	58	34.97	8.89	68.64	11.59
ASO	142	34.67	8.41	68.51	12.04

B. New examining methods and procedures

In light of the ongoing Covid 19 pandemic, the University changed the examinations to an open-book format and rolled out Inspera, a new online examinations platform. An additional 30 minutes was added on to the exam duration to allow candidate the technical time to download and submit their examination papers via Inspera.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

The department intends to hold in person exams in Trinity Term 2022.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 22nd March 2021 and the second notice on the 30th April 2021.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

- Barbara Galinska for their work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith for her help and support, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Prof Jelena Grbic and Prof John Billingham, for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 14th June and ended on Friday 25th June.

Mitigating Circumstances Notices to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers/assessors. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

The whole process of setting and checking the papers was managed digitally on WebLearn. Examiners adopted specific and detailed conventions to help with version checking and record keeping. This has worked very well.

Candidates accessed and downloaded their exam papers via the Inspera system at the designated exam time. Exam responses were uploaded to Inspera and made available to the Exam Board Administrator 25-33.5 hours after the exam paper had finished via One Drive.

The process for Marking, marks processing and checking was adjusted accordingly to fit in with the online exam responses.

Assessors were provided with the electronic mark sheets and had 1 to 2 weeks to mark the scripts and return the marksheets to their dedicated One Drive folders. A team of graduate checkers under the supervision of Barbara Galinska met virtually to script check the papers assigned to them. This included cross-checking the marksheets for each candidate against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the corners C_1 and C_2, which encode the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from C_1 to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between C_3 and C_2 and then again between $(0, 0)$ and C_3. Thus, the conversion of raw marks to USMs is fixed by the choice of the three corners C_1, C_2 and C_3. While the default y-values for these corners were given above and are not on the class borderlines, the examiners may opt to change those default values, e.g., to avoid distorting marks around class boundaries. The final choice of the scaling parameters is made by the examiners, guided by the advice from the Teaching Committee, considering the distribution of the raw marks and examining individuals on each paper around the borderlines.

In addition, in accordance with University's Examinations and Assessments Framework, once the proposed scaling parameters were agreed, the examiners compared the resulting medians with the average medians from the last three years. In all but one paper, the current year was within the safeguard interval and for one paper one scaling corner was adjusted.

The final resulting values of the parameters that the examiners chose are listed in Table 3.

Table 3: Parameter Values

Paper	C1	C2	C3
A0	(36.2,72)	(22,57)	(11.32,37)
A1	(43.8,72)	(30.3,57)	(17.41,37)
A2	(76,72)	(38.5,57)	(22.12,37)
A3	(43.2,71)	(24,57)	(12,37)
A4	(36.6,72)	(19.5,57)	(9.82,37)
A5	(44,70)	(29,60)	(20,51)
A6	(40.6,72)	(28.6,57)	(16.43,37)
A7	(34.2,72)	(17.7,57)	(10.17,37)
A8	(31,72)	(16,57)	(9.19,37)
A9	(38.6,72)	(19.1,57)	(10.97,37)
A10	(45.4,72)	(34.9,57)	(20.05,37)
A11	(40,70)	(22.5,57)	(12.93,37)
ASO	(40,70)	(23.1,57)	(13.27,37)

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
88.5	1	1	0.7
87.9	2	2	1.41
87.35	3	3	2.11
86.3	4	4	2.82
84.5	5	5	3.52
83.5	6	6	4.23
83.3	7	7	4.93
82.65	8	8	5.63
82.6	9	9	6.34
82.4	10	10	7.04
81.9	11	11	7.75
81.4	12	12	8.45
81.3	13	13	9.15
81	14	14	9.86
80.8	15	15	10.56
80.1	16	16	11.27
79.8	17	17	11.97
79.2	18	18	12.68
79	19	19	13.38
78.7	20	20	14.08
77.9	21	21	14.79
77.7	22	22	15.49
77.6	23	23	16.2
77.5	24	24	16.9
76.9	25	25	17.61
75.9	26	27	19.01
75	28	28	19.72
74.3	29	29	20.42
74.2	30	30	21.13
73.8	31	31	21.83
73.5	32	32	22.54
73.4	33	33	23.24
72.8	34	34	23.94
72.2	35	35	24.65
71.8	36	37	26.06
71.7	38	39	27.46
71.5	40	40	28.17
70.8	41	42	29.58
70.7	43	43	30.28
70.6	44	45	31.69
70.4	46	46	32.39
70.3	47	48	33.8

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
70.2	49	49	34.51
69.95	50	50	35.21
69.7	51	52	36.62
69.6	53	53	37.32
69.3	54	56	39.44
68.6	57	57	40.14
68.5	58	58	40.85
68.4	59	59	41.55
68	60	60	42.25
67.9	61	62	43.66
67.5	63	63	44.37
66.8	64	64	45.07
66.7	65	66	46.48
66.6	67	67	47.18
66.3	68	68	47.89
66.2	69	69	48.59
66.1	70	70	49.3
65.9	71	71	50
65.8	72	73	51.41
65.7	74	74	52.11
65.6	75	75	52.82
65.5	76	76	53.52
65.45	77	77	54.23
64.8	78	79	55.63
64.7	80	80	56.34
64.6	81	81	57.04
64.4	82	83	58.45
64.3	84	84	59.15
64.1	85	85	59.86
63.9	86	86	60.56
63.8	87	87	61.27
63.7	88	88	61.97
63.6	89	90	63.38
63.4	91	92	64.79
63.3	93	93	65.49
63.2	94	94	66.2
63.1	95	96	67.61
63	97	98	69.01
62.8	99	99	69.72
62.1	100	100	70.42
61.8	101	101	71.13
61.6	102	102	71.83

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
61.5	103	103	72.54
61.4	104	104	73.24
60.6	105	105	73.94
60.5	106	106	74.65
60	107	107	75.35
59.8	108	108	76.06
59.7	109	109	76.76
59.6	110	110	77.46
59.4	111	112	78.87
58.8	113	113	79.58
58.7	114	115	80.99
58.3	116	117	82.39
57.9	118	118	83.1
57.78	119	119	83.8
57.5	120	120	84.51
57.2	121	121	85.21
57.1	122	123	86.62
57	124	124	87.32
56.8	125	125	88.03
56.4	126	126	88.73
56.2	127	127	89.44
56	128	128	90.14
55.7	129	129	90.85
54.9	130	130	91.55
54.8	131	131	92.25
53.9	132	132	92.96
52.9	133	133	93.66
52.7	134	135	95.07
52.2	136	136	95.77
50.8	137	137	96.48
50.6	138	138	97.18
49.7	139	139	97.89
49	140	140	98.59
46.2	141	141	99.3
27.2	142	142	100

Recommendations for Next Year's Examiners and Teaching Committee

submission notifications for exams: a confirmation email should be sent to students automatically when they make an online submission in the examination platform (Inspera).

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page 9 shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2021			2020			2019		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	5	48	53	10	33	43	12	45	57
60–69	21	36	57	22	43	65	28	43	71
50–59	15	14	29	7	14	21	12	15	27
40–49	1	1	2	2	1	3	2	3	5
30–39	0	0	0	0	0	0	0	1	1
0–29	0	1	1	0	0	0	0	1	1
Total	42	100	142	41	91	132	54	108	162
Class	Percentage								
	2021			2020			2019		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	11.9	48	37.32	24.39	36.26	30.32	22.22	41.67	35.19
60–69	50	36	40.14	53.66	47.25	50.45	51.85	39.81	43.83
50–59	35.71	14	20.42	17.07	15.38	16.22	22.22	13.89	16.67
40–49	2.38	1	1.41	4.88	1.1	2.99	3.7	2.78	3.09
30–39	0	0	0	0	0	0	0	0.93	0.62
0–29	0	1	0.7	0	0	0	0	0.93	0.62
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.90	14.90	6.11	123.00	0
Q2	16.75	17.09	5.37	43.00	1
Q3	15.12	15.28	5.07	118.00	2

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.37	19.53	4.21	112.00	1
Q2	15.63	15.63	4.38	54.00	0
Q3	19.60	19.60	3.23	118.00	0

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.05	15.17	5.45	110.00	1
Q2	14.49	14.49	6.24	70.00	0
Q3	13.76	13.76	5.09	91.00	0
Q4	17.38	17.38	5.73	128.00	0
Q5	14.30	14.39	5.34	88.00	1
Q6	15.63	15.63	5.94	78.00	0

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.49	16.49	5.53	53.00	0
Q2	18.19	18.19	5.26	54.00	0
Q3	16.87	16.85	5.82	60.00	1

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.41	15.57	5.95	90.00	1
Q2	14.72	14.73	4.60	110.00	0
Q3	12.51	12.69	5.34	36.00	1

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.48	19.48	5.67	82.00	0
Q2	15.76	15.98	5.79	53.00	1
Q3	16.52	16.52	5.90	31.00	0

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.4	17.4	3.83	45.00	0
Q2	17.51	17.51	2.57	49.00	0
Q3	17.74	17.74	4.90	50.00	0

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.91	13.39	5.67	41.00	2
Q2	13.09	13.09	4.64	32.00	0
Q3	14.09	14.10	5.51	31.00	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.28	13.41	4.77	79.00	1
Q2	11.48	11.55	4.12	105.00	1
Q3	13.51	13.8	5.04	70.00	2

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.65	15.65	5.09	60.00	0
Q2	16.32	16.32	5.34	50.00	0
Q3	13.57	13.7	5.14	60.00	1

Paper A10: Fluids and Waves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.33	20.33	3.08	36.00	0
Q2	19.35	19.35	4.05	31.00	0
Q3	22.6	22.6	2.30	5.00	0

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.78	18.08	5.22	50.00	1
Q2	16.34	16.34	4.54	47.00	0
Q3	18.74	18.74	5.87	19.00	0

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.37	20.37	4.87	60.00	0
Q2	18.13	18.26	4.56	39.00	1
Q3	18.56	18.56	5.94	9.00	0
Q4	15.8	15.8	6.83	5.00	0
Q5	17.69	17.69	4.91	68.00	0
Q6	15.39	15.55	5.59	40.00	1
Q7	13.75	13.75	3.06	28.00	0
Q8	14	14		1.00	0
Q9	15.26	15.26	3.44	34.00	0

D. Comments on papers and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Question 1: Almost all candidates attempted this question with good results.

Question 1 was by far the most popular question being selected by over 85% of the students. Part (a)(i) was done correctly by everyone.

There were a lot of mistakes in (a) (ii). The common point was the assumption that for another subspace W of V, W/U is a well-defined vector subspace of W/V. Almost everyone was able to prove that the map in (a) (iii) is injective and more than half constructed a concrete counter-example to prove it is not surjective.

Most of the students obtained at least 4/7 in part (b). It was surprising to see some mistakes in the computation of characteristic polynomial given the open book format of the exam. Once the characteristic polynomial was calculated, about half the students failed to distinguish the cases based on λ and μ to calculate the minimal polynomial.

Part (c) of the question was answered generally well. One nice solution was to consider the matrices of S and S' with respect to some bases of V and W and their corresponding duals. Once this is proven, the claim reduces to the fact that the row rank of a matrix equals its column rank. There was significant percentage of students who reached the correct answer, but important steps in the proof were not justified.

Question 2:

Question 2 was chosen by a third of the candidates; this was interesting to observe as arguably it is an easier question than Question 3. Part (a) was done very well by a large majority of the candidates. Most students had no trouble with part (b), but those who did not explain how they used the condition that the matrices are 6×6 lost a lot of marks for their carelessness. Part (c)(i) was hard but manageable. The following nice solution which does not rely on the binomial expansion of $(1+x)^{\frac{1}{3}}$ was found by several candidates. If $B = \lambda I + J$ is a basic

$n \times n$ Jordan block, it is enough to show that the Jordan Normal form of B^3 is $\lambda^3 I + J$. To this end, it is enough to show that the minimum polynomial of $B^3 - \lambda^3 I$ is precisely x^n if $\lambda \neq 0$. This is done by observing that

$$((\lambda I + J)^3 - \lambda^3 I)^{n-1} = (3\lambda^2 J + O(J^2))^{n-1} = (3\lambda^2)^{n-1} J^{n-1}$$

is non-zero whenever $\lambda \neq 0$.

Part (c)(ii) was done very well by almost everyone.

Question 3:

Questions 3 was also popular being chosen by about 75% of the students. In part (a), everyone was able to prove that the form is bilinear. Almost everyone who made the observation the surjective is equivalent to injective obtained full marks.

Part (b) of the question turned out to be quite tricky. In (i), most students were able to use part (a), but due to a lack of care failed to prove that the map $\varphi : U^{\perp} \to U^{\circ}$ is also surjective.

There were very few complete answers to part (ii). Most of them started from the Minkowski's space example given in the lecture notes. The most common mistake was assuming/trying to prove that the form is non-degenerate.

In part (c), a large majority of students proved that the form is bilinear and non-degenerate. For giving a counterexample, the students constructed a space U such that $\dim U = \dim W = \frac{1}{2}\dim(W \times W')$ and $U \subset U^{\perp}$. There were very few correct answers to the last question as most assumed that part (b) (ii) is correct and tried to argue that the form in part (c) is not symmetric.

A1: Differential Equations 1

Questions 1 and 3 were equally popular, while question 2 saw 50% less attempts in comparison. The standard of answers was also comparable between questions 1 3, while question 2 had a lower average. Students often lost some marks on the easy parts of the problems due to lack of care.

Question 1. Most students who attempted Q1 found parts 1(a) and (c) easier in comparison to part (b). Determining the region of unique determinability and showing/explaining why u_y is discontinuous proved difficult for many students. Students also seemed to lose easy marks by not paying attention to what was asked, e.g., in 1b(iv) where they had to say if the result agrees with theory or not writing limits where a solution is valid.

Question 2. In Q2 students mostly did very well on part (a). Some however described the difference between the proofs by saying one is more direct while the other one is more abstract without giving any proper details. This was not sufficient. Part b.i) was answered very well. Part b.ii) caused troubles: while most computed the bounds M and L correctly, many failed to note that the problem was not asking for range of h, k for which solutions are guaranteed to exist but only for which k the extra condition $Lh < k$ was already implied by the first condition $Mh \leq k$. Most found the solution trajectory in b.iii) but many had troubles with seeing if it was defined for all $x \in (0, 2)$ or not. Part c.i) was answered very well but most students produced a variation of the proof of Gronwall's inequality instead of applying to inequality itself to a suitable function. Part c.ii) was answered very well except some cases where students confused the parameter β with the initial point b.

Question 3. Q3 was the most popular question in this exam and everyone who attempted the question got at least 9 raw marks. Students were able to use online plotters or scientific computing software to check their intuition (a few students referenced desmos in their work). Overall, the question required confidence in dealing with phase portraits and classifications of critical points. Errors were most often due to either a mistake in computations, e.g., only finding one out of two critical points or wrong sense of direction of a centre. Some marks were also routinely lost when students did not provide any justification for their results, e.g., in b.iii). Some students found showing that there are closed trajectories around $(0, \beta)$ in a.ii) difficult.

A2: Metric Spaces and Complex Analysis

Question 1. Candidates answered the bookwork well (a)(i), and most candidates obtained some or major marks for (a)(ii). While some candidates could not see that the space (1) is not path-connected but connected. Part (b) turns out to be the most challenging part for this question, many candidates have difficulty for applying epsilon-delta argument for proving the equivalence of open sets under different metrices. Most candidates did quite well for part (c) (i) and (ii) – good examples were produced for (ii) for justification. While many students didn't give a complete answer to part (c)(iii) – probably unable to come up a counterexample in the limited time, though there are plenty of easy and simple examples known from power series.

Question 2. Most candidates who attempted this question got most of the marks for part (a) but some missed one or two marks for proving the uniform convergence where a special partition is needed. For part (b), most candidates realised (ii) is just the statement that a continuous function on a compact space is uniformly continuous, so good proofs were constructed. But still a few candidates saying (ii) is just the definition of continuity, and therefore credits could not be awarded then. For part (c), most students can argue the impossibility by using connectedness, while some students try to use compactness but failed to prove the claim (c). Part (d) turns out to be the most challenging part for this question. Again many students are not comfortable using eplison-delta definition to prove the equivalence of (1) and (2) in part (c)(i). (ii) should be easy for those attempted, can be done by using the fact that the preimage of an open (closed) set under a continuous map is open (closed). For part (c)(iii) while some students, again here, have difficulty to argue properly by using eplison-delta definition.

Question 3. Attempted by slightly less than two-thirds of candidates. Most had little trouble with (a) subparts (i) — (iii). Subpart (iv) was much more of a challenge, though many picked up the difference made in the lecture notes between being holomorphic and complex differentiable. Part (b) was quite well done, with candidates spotting a couple of different approaches to the first part and some adapting standard counterexamples for the second part. There were a number of different proofs in (c) found, my favourite among them being an application of (b) (surprisingly, not actually the intended solution). Overall the question seemed very discerning with a broad range of marks achieved.

Question 4. This was a popular question, attempted by 90% of the candidates. For part (a), almost all candidates considered the right path integral, but many failed to realise that an application of Cauchy's formula was necessary to complete the argument. Instead, some tried considering real parts, which doesn't work in this case. Part (b) was well done in average, but some candidates seem to be confused about the different kinds of singularities, and most didn't notice that $z = 0$ was a non-isolated singularity in subpart (iii). Those comfortable with contour integration did very well on part (c), as the choice of contour and the application of the estimation lemma (or Jordan's lemma) to the function $\frac{e^{iz}}{(z^2+a^2)(z^2+b^2)}$ was somewhat standard. A few candidates tried to work with the function $\frac{\cos(z)}{(z^2+a^2)(z^2+b^2)}$ and got the wrong result at the end, the reason being that the estimation lemma doesn't apply in this case. When $a = b$, some candidates simply computed a limit on the result concerning the case $a \neq b$, instead of reapplying a contour integration.

Question 5. Again attempted by somewhat less than two-thirds of candidates, For part (a) (i) most realised an elementary computation led to the expansions, rather than applying the general machinery. The uniqueness in subpart (a) (ii) caused some difficulties, but many students got (iii). There were several different contours one could choose in part (b), some less easy to work with than others, and this did lead to some students making false starts, or at least changing contour midway through. The most common omission was forgetting about the pole at z = 1 in the integrand. Again overall the question seemed to differentiate well, with candidates scoring from very low to full marks.

Question 6. Either students gave good answer to part (a) or had no idea how to answer (a)(ii), although the question itself, which is not a bookwork or a standard exercise, is obvious for its answer. While some students use a holomorphic mapping to parametrize a simply connected domain, which is not asked by the question but is still good. Good answers are produced for part (b) for the use of Mobius transformations and their properties, although a few students went through length computations which could be short. Part (c) is the most difficult part of this question. Most those who attempted this part constructed a proper Mobius transformation, then define a holomorphic branch of the square root function using log (cutting off the positive or negative part of the real – depending the Mobius transformation used) so achieve a good grade for c(i). Few candidates attempted other approaches but failed. Overall finding holomorphic branch with two branching points proves not easy for candidates, although for this question (c)(ii) can be answered quickly by using the construction in (c)(i). Part (d) is a standard exercise for constructing holomorphic mappings from moon shape domains to standard ones half plane and the unit disk. Most those who attempted this part were able to construct a Mobius transformation sending the domain to a domain between to lines, while few candidates failed to carry out the whole process and obtained only fraction of the marks allocated.

Long Options

A3: Rings and Modules

Question 1.

Q1 (a) (i) Nice answers adapted the proof that X is prime in $R[X]$ for an integral domain R. Marks were lost for failing to adapts from the integers to general PIDs. (ii) Direct calculation was long and for full marks needed to justify equating coefficients. Better to apply previous part. (b) (i) Uniqueness was sometimes missed. (ii) Mapping the identity to the identity was sometimes missed. (iii) Many claimed that being a field was such a property.

Question 2.

Q2 (c) Was approached either directly or via the Isomorphism Theorem. Many lost a mark by not checking the domain is well-defined. Proving surjectivity was the most challenging part of the question.

Question 3. Q3 (a) Omitted by many. A common error was to mistake $\mathbb{Z}/\langle 1\rangle$ for $\mathbb{Z}$. (b) For full marks it was necessary to show that that the lowest common multiple of d and h is dh. (c) For full marks it was necessary to prove the part of the CRT being used. (d) Done well. (e) Some tried to show that the order of the group of units of a finite field is square-free and use the previous part.

A4: Integration

The raw marks were quite low, largely because the questions were of non-standard form due to the exam being open-book. They were spread out more than usual for similar reasons. These effects were taken into account by the examiners in producing scaled marks.

Question 1. This question produced a good spread of marks. In part (a), a significant proportion of candidates claimed that the MCT could be used directly, which is not the case. Part (b) was probably the trickiest part, involving a delicate use of DCT for continuous parameters. Part (c) was probably the easiest part in principle, but a significant number of candidates asserted that the MCT for Series justified integrating the series for x^x term-by-term. The MCT for Series does not work in a simple way for this series, although it can be used in a less direct way.

Question 2. This question produced a good spread of marks but they were slightly on the low side. Rather few scripts covered all cases in part (a). Part (b) was fairly straightforward. Part (c) was quite hard, and very few candidates produced valid arguments for both subparts. Fubini's theorem (in the contrapositive) can be used for (i) and Tonelli's theorem for (ii).

Question 3. This question was intended to involve variants of bookwork, but all parts of the question, especially (c), turned out to be more difficult than expected.

A5: Topology

Question 1. was selected by the large majority of students, and they generally got better marks for it than for Questions 2 and 3. In part 1.a, surprisingly few students were able to correctly observe that there are five different cases for $[a, b) \cap [c, d)$, depending on the relative positions of the real numbers a, b, c, and d. Many boldly stated that $\emptyset \in \mathcal{B}$, and few were able to accurately state that $\emptyset$ is a union of elements of $\mathcal{B}$ (a union indexed over the empty set). Part f.i and f.ii were supposed to be the hardest but were actually, unfortunately, not much harder that the preceding parts.

Question 2. went reasonably well. In 2.a.ii, many people felt the need to connect the two definitions of $\mathbb{RP}^2$ given in the exercise to what they thought is the more standard definition of $\mathbb{RP}^2$, as a quotient of $\mathbb{R}^3 \setminus \{0\}$. A substantial set of students tried to construct a map $S^2/\!\sim\; \to D^2/\!\sim$, which is of course the wrong direction, and makes it almost impossible to prove that it's continuous, let alone a homeomorphism. In 2.c, a typical mistake was to assume that every open set in $X \times Y$ is of the form $U \times V$.

Question 3. was selected by rather few students, but was done reasonably well by those who selected it. In 3.a.ii, a majority of students stated that every neighbourhood of the point $(0, 0)$ is homeomorphic to a "+", which is of course false as such a neighbourhood could e.g. be disconnected. Part 3.b.iii was a trap and many fell for it, not realising that not all vertices are identified. Part 3.c.ii, had the unfortunate feature that it sometimes lead to a combinatorial explosion of cases. This was not my intention, and I fear that some students might have lost a lot of time going through all these cases instead of using the more conceptual Euler characteristic argument which I had tried to hint at.

A6: Differential Equations 2

There was a good balance between the questions. Question 1 was the least popular, but by a small margin, having been chosen by 55 students. Questions 2 and 3 were very comparable, with 64 and 63 students respectively. Moreover, the average scores for the 3 questions are all between 17.75 and 18 out of 25, showing a similar level of difficulty. For each student, there is also a very good correlation between the scores obtained in the two chosen questions, except for a few outliers, often explained by the fact that a student did not have the time finish the second question. Note that only a few students seem to have struggled with time limitations.

Question 1.: 1(a) was overall very well done, even if some of the justifications were missing. For 1(b), several students struggled to find the right pair of functions for the method of variation of parameters, often choosing wrong boundary conditions. Most of the students did well for the first part of 1(c), but others did not attempt, possibly due to time constraints. In the second part of 1(c), arguments were not always sufficiently solid.

Question 2: 2(a) went well overall, even if most students did not consider the case $a = 0$. For 2(b), some students made calculations mistakes and did not find the right relations. They also often did not properly identify the solution in the general case. For question 2(c), the arguments were often not sufficient. Many students struggled to solve the case $a = 2k$. For 2(d), several students failed to identify the closed form solutions.

Question 3: 3(a) went very well, except for a few students who did calculation mistakes. The first part of 3(b) also went well, even if several students failed to identify the regions where each term dominates. The second part of 3(b) was a bit weaker, as the arguments were often unclear. Question 3(c) went well overall, even if several students made mistakes when matching the expansions and finding the composite solution.

A7: Numerical Analysis

Question 1.

Question 1 was popular and answered by most candidates. There was no question that was completely bookwork—and even 1(a) required some thinking, and some missed the key observation that n+1 points is enough to determine a polynomial interpolant uniquely.

b(ii) is an interesting question that can be answered using a technique presented in class, but in a different context. Many candidates seemed to find it challenging. The final questions c(ii) and especially c(iii) were intended to be very challenging.

Question 2.

Question 2 was also attempted by many candidates.

(a) checks basic understanding of the SVD. In the cost analysis, some included the cubic cost of finding the SVD; this is given and hence the required cost should be lower (quadratic).

b(iii) was stated in a somewhat tricky manner (which also served as a hingt for b(ii)), and naturally many incorrectly stated that a low-rank approximation is unique.

(c) can be solved using a technique for tightening Gerschgorin's bound, which was seen in a problem sheet, but the context was rather different and most failed to see the connection.

Question 3.

Question 3 was attempted by more than half the candidates.

(a)(i) is a standard problem that uses Taylor's theorem, but it also needs a simple bound for y, which some failed to see. (a)(ii) is lengthier but not harder.

In b(ii), many did not specify which sign choice should be chosen.

b(iii) asks candidates to use a linear algebra result from earlier in the course; an alternative is Newton's method, which is more common, but here the aim is to test the understanding of polynomial rootfinding via eigenvalues in a new context.

The final part (c) is not as difficult as one might imagine–many who made a serious attempt got a good mark. Some nonetheless did not make an attempt, perhaps thinking it would be difficult as it is the final problem.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

The majority of candidates attempted questions 1 and 2, with just a small number (14%) attempting question 3. In general all questions were well done, with Q3 scoring the highest, and Q2 the lowest. Detailed comments for each question are as follows.

Question 1 Part 1(a) was well done. Some candidates presented overly elaborate arguments for the shape of the streamlines for $\Omega > 1$ and $\Omega < 1$. Part (b)(i) was well done. In part (b)(ii) many candidates incorrectly computed the residues of $(dw/dz)^2$ corresponding to $z = 0$. Specifically, candidates ignored the contribution coming from the product of the $-U/z^2$ term in dw/dz with contributions from the $-1/b(1 - z/b)$ and $-b/(1 - bz)$ terms. Some candidates misunderstood "force per unit length", dividing the force obtained by 2π. Part (iii) was well done with the majority of candidates showing the force changes direction as b increases.

Question 2 Part (a)(i) was well done. In (a)(ii) some candidates did not consider a point in the z plane to show that the mapping is to the upper half ζ plane. Some candidates did not argue why $z = x, x \geq 1$ mapped to $\Im\zeta = 0$, $\Re\zeta \geq 2$ (and similarly for $z = x, x \leq -1$). In (a)(iii) some candidates did not ensure the mapping was single valued. Part (b) was in general very well done. A few candidates incorrectly used c rather than $g(c)$ as the location of the vortex in the ζ plane. In (c)(i) some candidates did not compute dw/dz accurately, forgetting that ζ is a function of z. In (c)(ii) the main error came when candidates incorrectly neglected any contribution to the vortex velocity from the term in dw/dz proportional to $1/(g(z)-g(c))$. Instead, candidates assumed this can be neglected when removing the contribution to the velocity due to the vortex itself. Some errors came in the direction the vortex will start to move, related to incorrectly computing the expression for the the instantaneous vortex velocity.

Question 3 This was well done overall. In (b)(ii) some students did not give a correct physical interpretation of $c = c^*$. There were some errors in the sketching.

A11: Quantum Theory

Question 1 Question 1 looks at a quantum particle on a circle (similar to a particle in a box), and was attempted by the majority of candidates. A common error in part (a) was not realizing that it is the periodic boundary condition that leads to n being an integer, while in part (b) many candidates didn't check that the wave function is normalized. Otherwise answers to these parts tended to be very good, and largely complete. Part (c) differentiated between candidates the most, with either largely complete answers, or minimal attempts. Part (iii) is the only part that required a computation, and is similar to an example on a problem sheet. Only a few candidates correctly answered the very last part.

Question 2 Question 2 examined raising and lowering operators for a quantum particle in two dimensions. Part (a)(i) was correctly answered by almost all candidates. Part (a)(ii) is best proved using induction on n, which many candidates did well, but some didn't notice that one needs the $n = 1$ result in the inductive step. The very last part can be answered straightforwardly by taking the adjoint, although only a small number of candidates noticed this. Part (b) was generally answered well, although some candidates made the problem longer than necessary by writing the number operator back in terms of raising and lowering operators, rather than using the results of part (a). Many answered (c)(i) correctly, but very few candidates got anywhere with part (c)(ii), perhaps partly due to time. There were a small handful of completely correct answers, however.

Question 3 Question 3 was the least popular question, although those who attempted it tended to do well. Part (a) concerns the spin 1/2 representation of angular momentum, and was generally very well answered. The most common error was not explaining why φ has the same J^2 eigenvalue as ψ. Part (b) looks at a two-state quantum system, and parts (i) and (ii) are similar to an example in the lecture notes. Perhaps as such, they were generally well answered. Part (iii) caused the most problems. While the correct probability was given, many candidates didn't give a proper explanation of the limit; for those that did, l'Hôpital's rule was a popular method. Any correct comment on the physical interpretation was accepted for a mark, although this clearly baffled some candidates. Pleasingly, one candidate noticed this is an instance of the Quantum Zeno Effect, although that certainly wasn't necessary to obtain the mark.

Short Options

ASO: Q1. Number Theory

Many candidates attempted this question. The standard of answers was generally good. Answers to the first part of the question were usually correct except for some candidates making a mistake in 1a. Part 2 and 3 of the question had many good answers with many different methods proofs. Part 4 of the question was completed successfully by most candidates, even if they did not mange parts 2 and 3.

ASO: Q2. Group Theory

This question was about applications of the Sylow theorems.

There were 47 attempts, of which 28 answers were in the 18-25 range, and 13 in the 13-17 range.

Most candidates showed a reasonable proficiency with how to use Sylow's theorems. In part (b) several failed to realise that an extension need not be a semidirect product, and hence gave incomplete answers.

Some of the arguments in part (c) were too sketchy, especially in showing that the Sylow subgroups commuted. Several answers were unnecessarily complicated in showing uniqueness of the Sylow subgroups, using counting arguments when in fact it followed easily from the third part of Sylow.

Part (d) was mostly quite well done, with most candidates understanding exactly what conditions were needed for the arguments of (c) to work. Part (e), as expected, proved more challenging, and many failed to justify the existence of a non-Abelian group. However several candidates did give very good answers here, explicitly constructing a nontrivial homomomorphism from H to Aut (N).

ASO: Q3. Projective Geometry

11 students attempted this question. There were several essentially complete solutions. Some people stumbled on the first part of (b); in particular, one type of attempted solution confused points in projective space with representing vectors, and projective transformations with representing linear maps, aiming to use linearity in a naive (and incorrect) way. The second part of (c) caused some problems for some, stemming perhaps from time pressure.

ASO: Q4. Introduction to Manifolds

Part a) was answered well by most candidates, with some students losing marks because they did not provide sufficient justifications for their assertions. Most candidates did well on part b), but part c) was more challenging. Nevertheless a couple of students did provide a complete solution, and a number noticed that the result followed from theorems established in complex analysis.

ASO: Q5. Integral Transforms

Part (a) was generally well answered, with most candidates obtaining high marks. While some revision of the reverse triangle inequality may have been useful in answering 5(a)(i)(1), most candidates successfully justified that the translation still had finite support. Several candidates failed to consider the case of $a = 0$ in 5(a)(i)(2), but 5(a)(i)(3) was generally the weakest part of the question, with many candidates not identifying that $a > 1$ causes $\phi_3(x)$ to have finite support.

The most common mistake in 5(b) was in omitting constants of integration. Many candidates successfully applied the Laplace transform to the ODE to obtain a differential equation for $\bar{f}_1$ but missed the constant of integration when solving (or assumed that it must equal 0). Those candidates who used the hint to find the constant of integration were generally able to use the convolution theorem to find $(f_0 * f_1)(x)$, although some candidates struggled with the inversion.

5(c) was generally well done by those who successfully applied the Fourier transform to the PDE, although some candidates struggled with the $\frac{\partial}{\partial x}(xu)$ term. Those who found the correct transformed equation were generally able to use the hint to obtain the form of $b(t)$, although sign errors were not uncommon here. The inversion was generally well done, but a couple of candidates misplaced the constants in the equation (possibly due to misreading the hint as giving the formula for the inverse of e^{-cs^2}).

ASO: Q6. Calculus of Variations

Overall the calculus of variations question seemed to work well despite the complications due to online exams and the open book format. Most students answered part (a) and (b) well, which were close to content seen in the lectures but nevertheless showed that most candidates had absorbed the basic ideas of the course. Part (c) was a longer more technical question which caused more difficulties, and was effective at separating the stronger answers from the weaker ones. It was pleasing that most candidates seemed to roughly appreciate what needed to be done even if they failed to execute the calculations completely. Part (d) had a slightly different style, and this tripped up some of the otherwise strong answers. Several candidates were confused by the distinction between being symmetric with respect to an artificial parameter and the curve itself being symmetric. The question would probably have been too long if not in open book format.

ASO: Q7. Graph Theory

Almost everyone could give the definitions of a minimum cost spanning tree (MCST) and a shortest paths tree. About 3/4 of candidates could compute the MCST in the example in (a), although there were quite a few errors. Almost all candidates could give a graph where the MCSTs and shortest paths trees necessarily differ. The uniqueness of the MCST in question (c)(i) where all edges have distinct weights was done well: it was a problem on an example sheet. Candidates found questions (c)(ii) and (c)(iii) more challenging. Around half of all candidates could give partial answers for (c)(ii), although few could give an accurate proof. What made (c)(ii) challenging was that it is easier to prove a more general statement, that distinct subsets of the edge set have distinct weights. Only a handful of candidates could answer (c)(iii); the solutions that were provided were quite original.

There was a missing hypothesis in part (c): the question should have assumed that G is connected. Without connectedness, the graph G does not have a MCST or a shortest paths tree. As the hypothesis of connectedness was obviously required, this omission caused no candidates any difficulties.

Overall, the question seems to have been quite successful. Every candidate was able to demonstrate some understanding of graph theory. But the harder parts at the end of the question could differentiate between the more able candidates.

ASO: Q8. Special Relativity

(a) and (b) are answered well by the students. One made reasonable progression (c). Students still are not really used to the concept of 4-velocity.

ASO: Q9. Modelling in Mathematical Biology

The first part of this question was done very well, but all candidates struggled with the last part of (b) meaning that no candidate scored full marks overall. Detailed comments:

(a) (i) Very few candidates could explain the meaning of the parameter b as a measure of competition at high population densities. (a) (iii) This question required candidates to take the analysis from (ii) but many repeated the analysis and some got it wrong the second time (having got it correct for (ii)). (a) (iv) In some cases candidates simply drew arrows on the sketch but did not give any written explanation of what these arrows represented. (b) (ii) The idea here was to use a Taylor expansion but many candidates did not do this. Surprisingly, a number of candidates could not write down a Taylor expansion for a function of two variables. (iii) Here, crucially, it was important to show that the steady lost stability at this point, in other words, it was stable up until this point. No candidate recognised this. A number showed that a period 6 oscillation occurs at this point but this does not prove that this point is where a bifurcation occurs. Given the difficulties experienced by all candidates towards the end of this question, a slight readjustment of the mark scheme was implemented.

E. Names of members of the Board of Examiners

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  Dr Neil. Laws
  Prof. Konstantin Ardakov
  Prof. Alan Lauder
  Prof. Zhongmin Qian
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